A FORTRAN PROGRAM FOR CALCULATING THE FREQUENCIES AND MODE SHAPES OF AN ARBITRARY SHAPE CYLINDRICAL TYPE SHELL BY UTILIZING A FINITE ELEMENT SOLUTION

BY
GEORGE GEORGOPOULOS
DEPARTMENT OF MECHANICAL AND AEROSPACE ENGINEERING
CENTER FOR ACOUSTICAL STUDIES
NORTH CAROLINA STATE UNIVERSITY
RALEIGH, NORTH CAROLINA

LARRY H. ROYSTER
DEPARTMENT OF MECHANICAL AND AEROSPACE ENGINEERING
CENTER FOR ACOUSTICAL STUDIES
NORTH CAROLINA STATE UNIVERSITY
RALEIGH, NORTH CAROLINA

TECHNICAL REPORT NO 24

SEPTEMBER, 1972

UNC

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)	2a. REPORT SECURITY CLASSIFICATION
Center for Acoustical Studies Mechanical and Aerospace Engineering Department NCSU, Raleigh, N. C. 27607	2b GROUP

3. REPORT TITLE

A Fortran Program For Calculating The Frequencies And Mode Shapes Of An Arbitrary Shape Cylindrical Type Shell By Utilizing A Finite Element Solution

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical Report 1972

5. AUTHOR(S) (Last name, first name, initial)

Royster, Larry H. Georgopolos, George

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September, 1972	35	2

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
NONR 486(11) b. PROJECT NO. NR 185-551 c. d.	Technical Report No. 24 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES

Unlimited

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY

13. ABSTRACT

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This report describes a Fortran IV program capable of obtaining a finite element numerical solution to the free axisymmetric vibrations of a cylindrical type shell of arbitrary shape.

DD FORM 1 JAN 64 1473

UNC

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Vibrations						
Class IV						
Flextensional						
Finite Element						
Eigenvalues						
Fortran Model						

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ABSTRACT

This report describes a Fortran IV program capable of obtaining a finite element numerical solution to the free axisymmetric vibrations of a cylindrical type shell of arbitrary shape.

INTRODUCTION

Present flextensional designs may be grouped into one of five different classes (1). In order to derive a mathematical model for any of the above mentioned flextensional transducers, it is essential to determine the vibrational characteristics of the shell itself. A finite difference approach for each shell's dynamic response has been investigated at the Center for Acoustical Sutides in the Department of Mechanical and Aerospace Engineeering at North Carolina State University. An attempt in improving the widely used finite difference method led to the present investigation of the finite element method which is widely used in determining stresses and strains in Aerospace Research and Development.

The program utilizes a finite element analytical model to calculate the total Mass and Stiffness matrices of an arbitrary shape cylindrical type shell (2). After the total Mass and Stiffness matrices are calculated, displacement boundary conditions are introduced by removing appropriate rows and columns corresponding to points of the shell which are rigidly restrained from motion. Finally, subroutine NROOT is used for the numerical solution of the shell's free axisymmetric vibration matrix equation, and the eigenvalues, frequency coefficients, natural frequencies, and eigenvectors are then printed out.

PROGRAM UTILIZATION

The program is written in Fortran IV (G or H level) to be used on an IBM 370/165 computer. The storage capacity required depends upon the number of finite elements desired. As an example, the sample program presented in this paper having a total number of 6 elements requires a storage of 100 K for the evaluation of the stiffness and mass matrices of each element. These numbers are stored on a disk. In addition, a storage of 160 K, together with a time parameter of $2\frac{1}{2}$ minutes is required for the main program for a complete run. A typical element geometry and coordinate notation is given in figure 1. A geometrical element assembly is given in figure 2.

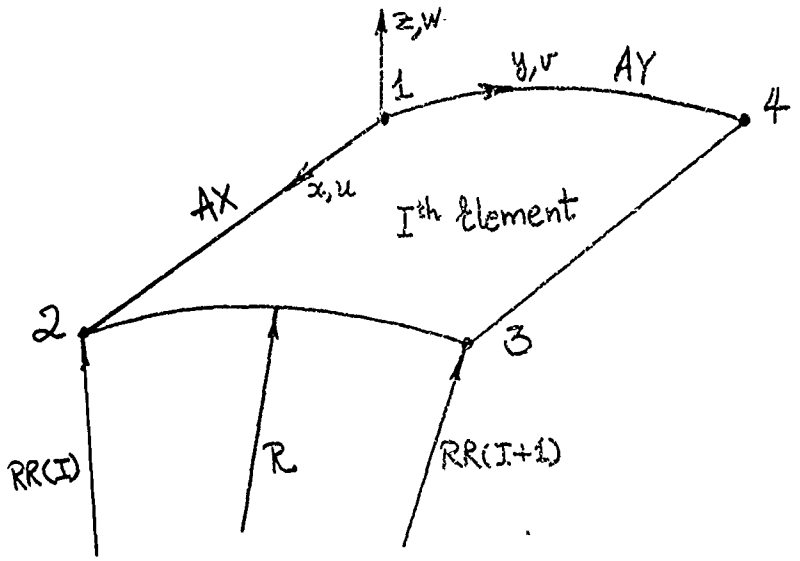

Figure 1. Individual finite element.

$$R = RR(I) + (RR(I+1) - RR(I))\, y/AY$$

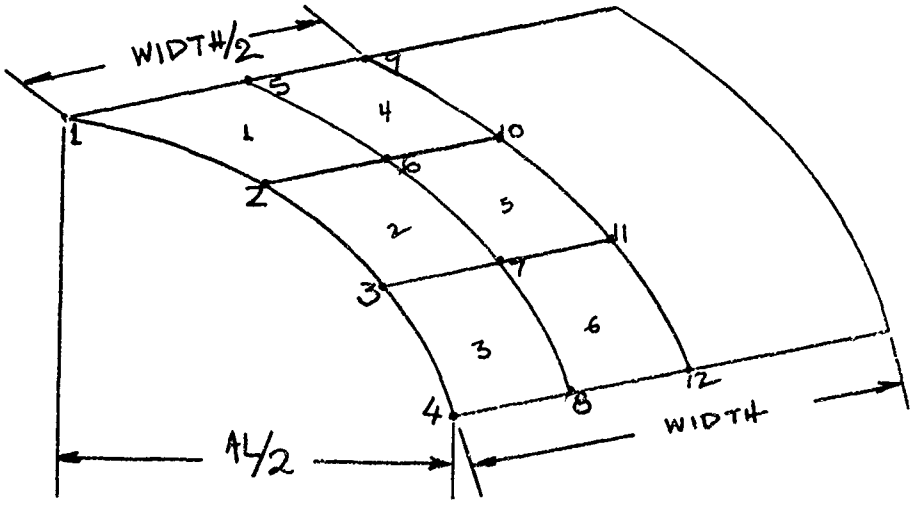

Figure 2. Finite element assembly of a shell.

Displacement vector δ_i per node is given as follows:

$$\delta_i = u_i, v_i, w_i, \frac{\partial u_i}{\partial x}, \frac{\partial v_i}{\partial x}, \frac{\partial w_i}{\partial x}, \frac{\partial u_i}{\partial y}, \frac{\partial v_i}{\partial y}, \frac{\partial w_i}{\partial y}$$

The program presented in Appendix A was written for the Class IV flextensional transducer shell, but it can be easily changed so it can be used for the case of any other similar type shell. Before using this particular program, it must be noted that all element stiffness and mass matrices are evaluated outside the main program. Then they are put on a disk from which the main program picks them up for total mass and stiffness formulation and final dynamic solution. It is necessary to use proper JCL cards. The program instruction in Appendix A must be read very carefully before running the program. A set of data cards is needed for the element notation and final mass and stiffness formulation. The program presented in Appendix A was written for figure 1. element notation and figure 2. element combination. All elements to form data for set 1. must be numbered according to figure 1 and figure 2.

ELEMENT NO. 1	5 1 2 6	SET 1
ELEMENT NO. 4	9 5 6 10	
ELEMENT NO. 2	6 2 3 7	
ELEMENT NO. 5	10 6 7 11	
ELEMENT NO. 3	7 3 4 8	
ELEMENT NO. 6	11 7 8 12	

The boundary conditions are:

$W_{y_i} = 0 \quad i = 1, 4, 5, 8, 9, 12$

$v_i = 0 \quad i =$ "

$v_{x_i} = 0 \quad i =$ "

$u_i = 0 \quad i = 9, 10, 11, 12$

$W_{y_i} = 0 \quad i =$ "

$W_{x_i} = 0 \quad i =$ "

These are applied to the corresponding vector index notation

	u	v	w	v_x	v_x	W_x	u_y	v_y	W_y
Node 1	1	(2)	3	4	(5)	6	7	8	(9)
Node 2	10	11	12	13	14	15	16	17	18
Node 3	19	20	21	22	23	24	25	26	27
Node 4	28	(29)	30	31	(32)	33	34	35	(36)
Node 5	37	(38)	39	40	(41)	42	43	44	(45)
Node 6	46	47	48	49	50	51	52	53	54
Node 7	55	56	57	58	59	60	61	62	63
Node 8	64	(65)	66	67	(68)	69	70	71	(72)
Node 9	(73)	(74)	75	76	(77)	(78)	(79)	80	(81)
Node 10	(82)	83	84	85	86	(87)	(88)	89	90
Node 11	(91)	92	93	94	95	(96)	(97)	98	99
Node 12	(100)	(101)	102	103	(104)	(105)	(106)	107	(108)

Therefore Set 3, correspdonding to the total number of the above mentioned 30 boundary conditions (NBC=30), is easily formed as follows:

2 - 5 - 9 - 20 - 32 - 36 - 38 - 41
45 - 65- 68- 72 - 73 - 74 - 77 - 78
79- 81- 82- 87 - 88 - 91 - 96 - 97
100-101-104-105- 106- 108

} SET 3

In order to form Set 2, the vector index notation is used again, but the circled numbers that correspond to the boundary conditions are omitted. Therefore, the vector index notation is as follows

Node 1	1	_	(2)	3	_	(4)	5	6	_
Node 2	(7)	8	9	10	11	12	13	14	15
Node 3	16	17	18	19	20	21	22	23	24
Node 4	25	_	(26)	27	_	(28)	29	30	_
Node 5	(31)	_	(32)	33	_	(34)	35	36	_
Node 6	(37)	38	39	40	41	42	43	44	45
Node 7	46	47	48	49	50	51	52	53	54
Node 8	55	_	(56)	57	_	(58)	59	60	_
Node 9	_	_	(61)	62	_	_	_	(63)	_
Node 10	_	(64)	65	66	_	_	_	(68)	69
Node 11	_	(70)	71	72	73	_	_	(74)	75
Node 12	_	_	(76)	77	_	_	_	(78)	_ (79)

Then Set 2 is formed as follows:

2 - 4 - 7 - 26- 28- 31- 32- 34
37- 56- 58- 61- 61- 61- 63- 63
63- 64- 64- 68- 68- 70- 74- 74
76- 76- 78- 78- 78- 79

} SET 2

Then the data cards consist of Set 1- Set 2- Set 1- Set 3 in that order. Furthermore if a coordinate transformation is decired, then matrix AKL (I,J), card MAIN 85, should be assigend properly and also cards DISK 27 and MAIN 266, 267, 268 should be removed from the program. In addition, card MAIN 54 should be equal to 1.

FINAL REMARKS

A 5 point numberical integration is carried out for each element's stiffness and mass matrix by using abscissas and weight functions of Legendre polynomials. After combining all the individual stiffness and mass matrices and introducing boundary conditions, the equation of motion for the shell is written as follows:

$$[[K]-\lambda[M]]\{\delta\}^e = \{0\}$$

where

$[K]$ = Total stiffness matrix

$[M]$ = Total mass matrix

$\{\delta\}^e$ = Nodal vector

$\lambda = E/(2 \cdot RHO \cdot (1-POISS^2) \cdot \omega^2)$

RHO = Density

E = Modulus of Elasticity

$POISS$ = Poisson's Ration

Since subroutine NROOT is used to determine λ, the largest λ is determined first, and as a result, the lowest natural frequency is calculated first and so is the corresponding eigenvector.

REFERENCES

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2. Georgopoulos, George and L. H. Royster, "Development of a Finite Element Model for the Class IV Flextensional Underwater Transducer Shell", Technical Report No. 23, Department of Mechanical and Aerospace Engineering, North Carolina State University, Raleigh, N. C., September, 1972.

APPENDIX A

```
      DIMENSION AKK(36,36)                                          DISK    1
C     AKK(NF,NF)                                                    DISK    2
C     NF=DEGREES OF FREEDOM PER ELEMENT                             DISK    3
C     MF=NUMBER OF CURVELINEAR COORDINATES                          DISK    4
C     NM=NUMBER OF ELEMENTS IN X-DIRECTION (WIDTH)                  DISK    5
C     NELEM=NUMBER OF ELEMENTS                                      DISK    6
C     POISS=POISSON'S RATIO                                         DISK    7
C     THICK=THICKNESS (IN)                                          DISK    8
C     AX=LENGTH OF THE ELEMENT IN X-DIRECTION (IN)                  DISK    9
C     AY=LENGTH OF THE ELEMENT IN Y-DIRECTION (IN)                  DISK   10
      MF=3                                                          DISK   11
      NF=36                                                         DISK   12
      NM=2                                                          DISK   13
      NELEM=6                                                       DISK   14
      IJ=NELEM/NM                                                   DISK   15
      AX=0.75                                                       DISK   16
      AY=1.542                                                      DISK   17
      THICK=0.4                                                     DISK   18
      POISS=0.334                                                   DISK   19
      NAK=0                                                         DISK   20
    1 NAK=NAK+1                                                     DISK   21
      DO 2 I=1,IJ                                                   DISK   22
      CALL GSLGN(NAK,I,MF,NF,AX,AY,THICK,POISS,AKK)                 DISK   23
      WRITE(8,3)  (((AKK(I1,J1),J1=1,NF),I1=1,NF))                  DISK   24
    3 FORMAT(/,4E15.7)                                              DISK   25
    2 CONTINUE                                                      DISK   26
      IJ=1                                                          DISK   27
      IF (NAK.EQ.1) GO TO 1                                         DISK   28
      CALL EXIT                                                     DISK   29
      END                                                           DISK   30


      SUBROUTINE GSLGN(NAK,I,MF,NF,AX,AY,THICK,POISS,AKK)           GSLG    1
```

```
      DIMENSION AKK(36,36),AKL(36,36),AKM(36,36),BF(36,36),AA(5),H(5),RRGSLG    2
     1(4)                                                                GSLG    3
C     AKK(NF,NF)                                                         GSLG    4
C     AKL(NF,NF)                                                         GSLG    5
C     AKM(NF,NF)                                                         GSLG    6
C     BF(NF,NF)                                                          GSLG    7
C     AA(NX)                                                             GSLG    8
C     H(NX)                                                              GSLG    9
C     RR(IMAX+1)                                                         GSLG   10
C     IF NAK=1 THEN AKK(NF,NF)=STIFFNESS                                 GSLG   11
C     IF NAK=2 THEN AKK(NF,NF)=MASS                                      GSLG   12
C     D=E/(2*RHO*(1.0-POISS*POISS))                                      GSLG   13
C     RR(I)=RADIUS OF CURVATURE AT I TH NODE (IN),DIMENSION OF RR(I)=    GSLG   14
C     NUMBER OF ELELEMENTS IN Y-DIRECTION+1                              GSLG   15
      RR(1)=4.70                                                         GSLG   16
      RR(2)=4.0                                                          GSLG   17
      RR(3)=3.50                                                         GSLG   18
      RR(4)=1.90                                                         GSLG   19
      R1=RR(I)                                                           GSLG   20
      R2=RR(I+1)                                                         GSLG   21
      NX=5                                                               GSLG   22
      NY=5                                                               GSLG   23
      NZ=5                                                               GSLG   24
      AA(1)=-0.90618                                                     GSLG   25
      AA(2)=-0.538469                                                    GSLG   26
      AA(3)=0                                                            GSLG   27
      AA(4)=0.538469                                                     GSLG   28
      AA(5)=0.90618                                                      GSLG   29
      H(1)=0.236927                                                      GSLG   30
      H(2)=0.478629                                                      GSLG   31
      H(3)=0.568889                                                      GSLG   32
      H(4)=0.478629                                                      GSLG   33
      H(5)=0.236927                                                      GSLG   34
      X1=0                                                               GSLG   35
      X2=1                                                               GSLG   36
      Y1=0                                                               GSLG   37
```

```
      Y2=1                                                              GSLG  38
      Z1=-THICK/2.0                                                     GSLG  39
      Z2=THICK/2.0                                                      GSLG  40
      IF (NAK.EQ.2) GO TO 10                                            GSLG  41
      DO 1 J1=1,NF                                                      GSLG  42
      DO 1 I1=J1,NF                                                     GSLG  43
      AKK(I1,J1)=0                                                      GSLG  44
1     CONTINUE                                                          GSLG  45
      DO 2 KL=1,NZ                                                      GSLG  46
      Z=AA(KL)*(Z2-Z1)/2.0+(Z2+Z1)/2.0                                  GSLG  47
      DO 3 J1=1,NF                                                      GSLG  48
      DO 3 I1=J1,NF                                                     GSLG  49
      AKL(I1,J1)=0                                                      GSLG  50
3     CONTINUE                                                          GSLG  51
      DO 4 K=1,NY                                                       GSLG  52
      Y=AA(K)*(Y2-Y1)/2.0+(Y2+Y1)/2.0                                   GSLG  53
      DO 5 J1=1,NF                                                      GSLG  54
      DO 5 I1=J1,NF                                                     GSLG  55
      AKM(I1,J1)=0                                                      GSLG  56
5     CONTINUE                                                          GSLG  57
      DO 6 L=1,NX                                                       GSLG  58
      X=AA(L)*(X2-X1)/2.0+(X2+X1)/2.0                                   GSLG  59
      CALL FSTIF(MF,NF,AX,AY,POISS,R1,R2,X,Y,Z,BF)                      GSLG  60
      DO 7 J1=1,NF                                                      GSLG  61
      DO 7 I1=J1,NF                                                     GSLG  62
      AKM(I1,J1)=AKM(I1,J1)+H(L)*BF(I1,J1)                              GSLG  63
7     CONTINUE                                                          GSLG  64
6     CONTINUE                                                          GSLG  65
      DO 8 J1=1,NF                                                      GSLG  66
      DO 8 I1=J1,NF                                                     GSLG  67
      AKL(I1,J1)=AKL(I1,J1)+H(K)*AKM(I1,J1)                             GSLG  68
8     CONTINUE                                                          GSLG  69
4     CONTINUE                                                          GSLG  70
      DO 9 J1=1,NF                                                      GSLG  71
      DO 9 I1=J1,NF                                                     GSLG  72
      AKK(I1,J1)=AKK(I1,J1)+H(KL)*AKL(I1,J1)                            GSLG  73
```

```
 9      CONTINUE                                                          GSLG  74
 2      CONTINUE                                                          GSLG  75
        GO TO 20                                                          GSLG  76
10      DO 11 J1=1,NF                                                     GSLG  77
        DO 11 I1=J1,NF                                                    GSLG  78
        AKK(I1,J1)=0                                                      GSLG  79
11      CONTINUE                                                          GSLG  80
        DO 12 K=1,NY                                                      GSLG  81
        Y=AA(K)*(Y2-Y1)/2.0+(Y2+Y1)/2.0                                   GSLG  82
        DO 13 J1=1,NF                                                     GSLG  83
        DO 13 I1=J1,NF                                                    GSLG  84
        AKL(I1,J1)=0                                                      GSLG  85
13      CONTINUE                                                          GSLG  86
        DO 14 L=1,NX                                                      GSLG  87
        X=AA(L)*(X2-X1)/2.0+(X2+X1)/2.0                                   GSLG  88
        CALL FMASS(NF,NF,AX,AY,R1,R2,X,Y,BF)                              GSLG  89
        DO 15 J1=1,NF                                                     GSLG  90
        DO 15 I1=J1,NF                                                    GSLG  91
        AKL(I1,J1)=AKL(I1,J1)+H(L)*BF(I1,J1)                              GSLG  92
15      CONTINUE                                                          GSLG  93
14      CONTINUE                                                          GSLG  94
        DO 16 J1=1,NF                                                     GSLG  95
        DO 16 I1=J1,NF                                                    GSLG  96
        AKK(I1,J1)=AKK(I1,J1)+H(K)*AKL(I1,J1)                             GSLG  97
16      CONTINUE                                                          GSLG  98
12      CONTINUE                                                          GSLG  99
20      DO 19 I1=1,NF                                                     GSLG 100
        DO 19 J1=I1,NF                                                    GSLG 101
        AKK(I1,J1)=AKK(J1,I1)                                             GSLG 102
19      CONTINUE                                                          GSLG 103
        RETURN                                                            GSLG 104
        END                                                               GSLG 105
```

```
      SUBROUTINE FSTIF(MF,NF,AX,AY,POISS,R1,R2,X,Y,Z,BF)                STIF    1
      DIMENSION DD(3,3),BB(3,36),BD(36,3),BF(36,36)                     STIF    2
C     BB(MF,NF)                                                         STIF    3
C     DD(MF,MF)                                                         STIF    4
C     BD(MF,MF)                                                         STIF    5
C     BF(NF,NF)                                                         STIF    6
C     DD(MF,MF)=E/(1.0-POISS*POISS)*DD(MF,MF)                           STIF    7
C     STIFF(NF,NF)=E/(1.0-POISS*POISS)*1/2*THICK*AX*AY*BF(NF,NF)        STIF    8
      R=R1+(R2-R1)*Y                                                    STIF    9
      DR=(R2-R1)/AY                                                     STIF   10
      DD(1,1)=1                                                         STIF   11
      DD(1,2)=POISS                                                     STIF   12
      DD(1,3)=0                                                         STIF   13
      DD(2,1)=POISS                                                     STIF   14
      DD(2,2)=1                                                         STIF   15
      DD(2,3)=0                                                         STIF   16
      DD(3,1)=0                                                         STIF   17
      DD(3,2)=0                                                         STIF   18
      DD(3,3)=(1.0-POISS)/2.0                                           STIF   19
      BB(1,1)=(-Y-6*X+6*X*Y+3*Y*Y+6*X*X-6*X*X*Y-2*Y*Y*Y)/AX             STIF   20
      BB(2,1)=0                                                         STIF   21
      BB(3,1)=(-X-6*Y+3*X*X+6*X*Y+6*Y*Y-2*X*X*X-6*X*Y*Y)/AY             STIF   22
      BB(1,2)=0                                                         STIF   23
      BB(2,2)=(1.0+Z/R)*(-X-6*Y+3*X*X+6*X*Y+6*Y*Y-2*X*X*X-6*X*Y*Y)/AY-Z*STIF   24
     1DR*(1.0-X*Y-3*X*X-3*Y*Y+3*X*X*Y+3*X*Y*Y+2*X*X*X+2*Y*Y*Y-2*X*X*X*Y-STIF   25
     12*X*Y*Y*Y)/(R*R)                                                  STIF   26
      BB(3,2)=(1.0+2*Z/R)*(-Y-6*X+6*X*Y+3*Y*Y+6*X*X-6*X*X*Y-2*Y*Y*Y)/AX STIF   27
      BB(1,3)=-6*Z*(-1.0+Y+2*X-2*X*Y)/(AX*AX)                           STIF   28
      BB(2,3)=(1.0-X*Y-3*X*X-3*Y*Y+3*X*X*Y+3*X*Y*Y+2*X*X*X+2*Y*Y*Y-2*X*XSTIF   29
     1*X*Y-2*X*Y*Y*Y)/R-6*Z*(-1.0+X+2*Y-2*X*Y)/(AY*AY)                  STIF   30
      BB(3,3)=-2*Z*(-1.0+6*X+6*Y-6*X*X-6*Y*Y)/(AX*AY)                   STIF   31
      BB(1,4)=1.0-Y-4*X+4*X*Y+3*X*X-3*X*X*Y                             STIF   32
      BB(2,4)=0                                                         STIF   33
      BB(3,4)=(-X+2*X*X-X*X*X)*AX/AY                                    STIF   34
      BB(1,5)=0                                                         STIF   35
```

```
 BB(2,5)=(1.0+Z/R)*(-X+2*X*X-X*X*X)*AX/AY-Z*AX*DR*(X-X*Y-2*X*X+2*X*STIF   36
1X*Y+X*X*X-X*X*X*Y)/(R*R)                                          STIF   37
 BB(3,5)=(1.0+2*Z/R)*(1.0-Y-4*X+4*X*Y+3*X*X-3*X*X*Y)               STIF   38
 BB(1,6)=-Z*(-4.0+4*Y+6*X-6*X*Y)/AX                                STIF   39
 BB(2,6)=AX*(X-X*Y-2*X*X+2*X*X*Y+X*X*X-X*X*X*Y)/R                  STIF   40
 BB(3,6)=-2*Z*(-1.0+4*X-3*X*X)/AY                                  STIF   41
 BB(1,7)=AY*(-Y+2*Y*Y-Y*Y*Y)/AX                                    STIF   42
 BB(2,7)=0                                                         STIF   43
 BB(3,7)=1.0-X-4*Y+4*X*Y+3*Y*Y-3*X*Y*Y                             STIF   44
 BB(1,8)=0                                                         STIF   45
 BB(2,8)=(1.0+Z/R)*(1.0-X-4*Y+4*X*Y+3*Y*Y-3*X*Y*Y)-Z*AY*DR*(Y-X*Y-2STIF   46
1*Y*Y+2*X*Y*Y+Y*Y*Y-X*Y*Y*Y)/(R*R)                                 STIF   47
 BB(3,8)=AY*(1.0+2*Z/R)*(-Y+2*Y*Y-Y*Y*Y)/AX                        STIF   48
 BB(1,9)=0                                                         STIF   49
 BB(2,9)=AY*(Y-X*Y-2*Y*Y+2*X*Y*Y+Y*Y*Y-X*Y*Y*Y)/R-Z*(-4.0+4*X+6*Y-6STIF   50
1*X*Y)/AY                                                          STIF   51
 BB(3,9)=-2*Z*(-1.0+4*Y-3*Y*Y)/AX                                  STIF   52
 BB(1,10)=(Y+6*X-6*X*Y-3*Y*Y-6*X*X+6*X*X*Y+2*Y*Y*Y)/AX             STIF   53
 BB(2,10)=0                                                        STIF   54
 BB(3,10)=(X-3*X*X-6*X*Y+2*X*X*X+6*X*Y*Y)/AY                       STIF   55
 BB(1,11)=0                                                        STIF   56
 BB(2,11)=(1.0+Z/R)*(X-3*X*X-6*X*Y+2*X*X*X+6*X*Y*Y)/AY-Z*DR*(X*Y+3*STIF   57
1X*X-3*X*X*Y-3*X*Y*Y-2*X*X*X+2*X*X*X*Y+2*X*Y*Y*Y)/(R*R)             STIF   58
 BB(3,11)=(1.0+2*Z/R)*(Y+6*X-6*X*Y-3*Y*Y-6*X*X+6*X*X*Y+2*Y*Y*Y)/AX STIF   59
 BB(1,12)=-6*Z*(1.0-Y-2*X+2*X*Y)/(AX*AX)                           STIF   60
 BB(2,12)=(X*Y+3*X*X-3*X*X*Y-3*X*Y*Y-2*X*X*X+2*X*X*X*Y+2*X*Y*Y*Y)/RSTIF   61
1-6*Z*(-X+2*X*Y)/(AY*AY)                                           STIF   62
 BB(3,12)=-2*Z*(1.0-6*X-6*Y+6*X*X+6*Y*Y)/(AX*AY)                   STIF   63
 BB(1,13)=-2*X+2*X*Y+3*X*X-3*X*X*Y                                 STIF   64
 BB(2,13)=0                                                        STIF   65
 BB(3,13)=(X*X-X*X*X)*AX/AY                                        STIF   66
 BB(1,14)=0                                                        STIF   67
 BB(2,14)=(1.0+Z/R)*(X*X-X*X*X)*AX/AY-Z*AX*DR*(-X*X+X*X*Y+X*X*X-X*XSTIF   68
1*X*Y)/(R*R)                                                       STIF   69
 BB(3,14)=(1.0+2*Z/R)*(-2*X+2*X*Y+3*X*X-3*X*X*Y)                   STIF   70
 BB(1,15)=-2*Z*(-1.0+Y+3*X-3*X*Y)/AX                               STIF   71
```

```
 BB(2,15)=(-X*X+X*X*Y+X*X*X-X*X*X*Y)*AX/R                                    STIF  72
 BB(3,15)=-2*Z*(2*X-3*X*X)/AY                                                STIF  73
 BB(1,16)=(Y-2*Y*Y+Y*Y*Y)*AY/AX                                              STIF  74
 BB(2,16)=0                                                                  STIF  75
 BB(3,16)=X-4*X*Y+3*X*Y*Y                                                    STIF  76
 BB(1,17)=0                                                                  STIF  77
 BB(2,17)=(1.0+Z/R)*(X-4*X*Y+3*X*Y*Y)-Z*AY*DR*(X*Y-2*X*Y*Y+X*Y*Y*Y)STIF  78
1/(R*R)                                                                      STIF  79
 BB(3,17)=(1.0+2*Z/R)*(Y-2*Y*Y+Y*Y*Y)*AY/AX                                  STIF  80
 BB(1,18)=0                                                                  STIF  81
 BB(2,18)=(X*Y-2*X*Y*Y+X*Y*Y*Y)*AY/R-Z*(-4*X+6*X*Y)/AY                       STIF  82
 BB(3,18)=-2*Z*(1.0-4*Y+3*Y*Y)/AX                                            STIF  83
 BB(1,19)=(-Y+6*X*Y+3*Y*Y-6*X*X*Y-2*Y*Y*Y)/AX                                STIF  84
 BB(2,19)=0                                                                  STIF  85
 BB(3,19)=(-X+3*X*X+6*X*Y-2*X*X*X-6*X*Y*Y)/AY                                STIF  86
 BB(1,20)=0                                                                  STIF  87
 BB(2,20)=(1.0+Z/R)*(-X+3*X*X+6*X*Y-2*X*X*X-6*X*Y*Y)/AY-Z*DR*(-X*Y+STIF  88
13*X*X*Y+3*X*Y*Y-2*X*X*X*Y-2*X*Y*Y*Y)/(R*R)                                  STIF  89
 BB(3,20)=(1.0+2*Z/R)*(-Y+6*X*Y+3*Y*Y-6*X*X*Y-2*Y*Y*Y)/AX                    STIF  90
 BB(1,21)=-6*Z*(Y-2*X*Y)/(AX*AX)                                             STIF  91
 BB(2,21)=(-X*Y+3*X*X*Y+3*X*Y*Y-2*X*X*X*Y-2*X*Y*Y*Y)/R-6*Z*(X-2*X*YSTIF  92
1)/(AY*AY)                                                                   STIF  93
 BB(3,21)=-2*Z*(-1.0+6*X+6*Y-6*X*X-6*Y*Y)/(AX*AY)                            STIF  94
 BB(1,22)=-2*X*Y+3*X*X*Y                                                     STIF  95
 BB(2,22)=0                                                                  STIF  96
 BB(3,22)=(-X*X+X*X*X)*AX/AY                                                 STIF  97
 BB(1,23)=0                                                                  STIF  98
 BB(2,23)=(1.0+Z/R)*(-X*X+X*X*X)*AX/AY-Z*AX*DR*(-X*X*Y+X*X*X*Y)/(R*STIF  99
1R)                                                                          STIF 100
 BB(3,23)=(1.0+2*Z/R)*(-2*X*Y+3*X*X*Y)                                       STIF 101
 BB(1,24)=-Z*(-2*Y+6*X*Y)/AX                                                 STIF 102
 BB(2,24)=AX*(-X*X*Y+X*X*X*Y)/R                                              STIF 103
 BB(3,24)=-2*Z*(-2*X+3*X*X)/AY                                               STIF 104
 BB(1,25)=(-Y*Y+Y*Y*Y)*AY/AX                                                 STIF 105
 BB(2,25)=0                                                                  STIF 106
 BB(3,25)=-2*X*Y+3*X*Y*Y                                                     STIF 107
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```
 BB(1,26)=0                                                                    STIF 108
 BB(2,26)=(1.0+Z/R)*(-2*X*Y+3*X*Y*Y)-Z*AY*DR*(-X*Y*Y+X*Y*Y*Y)/(R*R)STIF 109
 BB(3,26)=(1.0+2*Z/R)*(-Y*Y+Y*Y*Y)*AY/AX                                       STIF 110
 BB(1,27)=0                                                                    STIF 111
 BB(2,27)=(-X*Y*Y+X*Y*Y*Y)*AY/R-Z*(-2*X+6*X*Y)/AY                              STIF 112
 BB(3,27)=-2*Z*(-2*Y+3*Y*Y)/AX                                                 STIF 113
 BB(1,28)=(Y-6*X*Y-3*Y*Y+6*X*X*Y+2*Y*Y*Y)/AX                                   STIF 114
 BB(2,28)=0                                                                    STIF 115
 BB(3,28)=(X+6*Y-3*X*X-6*X*Y-6*Y*Y+2*X*X*X+6*X*Y*Y)/AY                         STIF 116
 BB(1,29)=0                                                                    STIF 117
 BB(2,29)=(1.0+Z/R)*(X+6*Y-3*X*X-6*X*Y-6*Y*Y+2*X*X*X+6*X*Y*Y)/AY-Z/STIF 118
1(R*R)*(X*Y+3*Y*Y-3*X*X*Y-3*X*Y*Y-2*Y*Y*Y+2*X*X*X*Y+2*X*Y*Y*Y)*DR  STIF 119
 BB(3,29)=(1.0+2*Z/R)*(Y-6*X*Y-3*Y*Y+6*X*X*Y+2*Y*Y*Y)/AX                       STIF 120
 BB(1,30)=-6*Z*(-Y+2*X*Y)/(AX*AX)                                              STIF 121
 BB(2,30)=(X*Y+3*Y*Y-3*X*X*Y-3*X*Y*Y-2*Y*Y*Y+2*X*X*X*Y+2*X*Y*Y*Y)/RSTIF 122
1-6*Z*(1.0-X-2*Y+2*X*Y)/(AY*AY)                                               STIF 123
 BB(3,30)=-2*Z*(1.0-6*X-6*Y+6*X*X+6*Y*Y)/(AX*AY)                               STIF 124
 BB(1,31)=Y-4*X*Y+3*X*X*Y                                                      STIF 125
 BB(2,31)=0                                                                    STIF 126
 BB(3,31)=(X-2*X*X+X*X*X)*AX/AY                                                STIF 127
 BB(1,32)=0                                                                    STIF 128
 BB(2,32)=(1.0+Z/R)*(X-2*X*X+X*X*X)*AX/AY-Z*AX*DR*(X*Y-2*X*X*Y+X*X*STIF 129
1X*Y)/(R*R)                                                                    STIF 130
 BB(3,32)=(1.0+2*Z/R)*(Y-4*X*Y+3*X*X*Y)                                        STIF 131
 BB(1,33)=-Z*(-4*Y+6*X*Y)/AX                                                   STIF 132
 BB(2,33)=(X*Y-2*X*X*Y+X*X*X*Y)*AX/R                                           STIF 133
 BB(3,33)=-2*Z*(1.0-4*X+3*X*X)/AY                                              STIF 134
 BB(1,34)=(Y*Y-Y*Y*Y)*AY/AX                                                    STIF 135
 BB(2,34)=0                                                                    STIF 136
 BB(3,34)=-2*Y+2*X*Y+3*Y*Y-3*X*Y*Y                                             STIF 137
 BB(1,35)=0                                                                    STIF 138
 BB(2,35)=(1.0+Z/R)*(-2*Y+2*X*Y+3*Y*Y-3*X*Y*Y)-Z*AY*DR*(-Y*Y+X*Y*Y+STIF 139
1Y*Y*Y-X*Y*Y*Y)/(R*R)                                                          STIF 140
 BB(3,35)=(1.0+2*Z/R)*(Y*Y-Y*Y*Y)*AY/AX                                        STIF 141
 BB(1,36)=0                                                                    STIF 142
 BB(2,36)=(-Y*Y+X*Y*Y+Y*Y*Y-X*Y*Y*Y)*AY/R-Z*(-2.0+2*X+6*Y-6*X*Y)/AYSTIF 143
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      BB(3,36)=-2*Z*(2*Y-3*Y*Y)/AX                                      STIF 144
      CALL GTPRD(BB,DD,BD,MF,NF,MF)                                     STIF 145
      CALL GMPRD(BD,BB,BF,NF,MF,NF)                                     STIF 146
      RETURN                                                            STIF 147
      END                                                               STIF 148


      SUBROUTINE FMASS(MF,NF,AX,AY,R1,R2,X,Y,BF)                        MASS   1
      DIMENSION AN(3,36),AS(36,3),BF(36,36)                             MASS   2
C     AN(MF,NF)                                                         MASS   3
C     AS(NF,MF)                                                         MASS   4
C     BF(NF,NF)                                                         MASS   5
C     MASS(NF,NF)=RHO*THICK*AX*AY*BF(NF,NF)                             MASS   6
      DO 1 I1=1,MF                                                      MASS   7
      DO 1 J1=1,NF                                                      MASS   8
      AN(I1,J1)=0                                                       MASS   9
    1 CONTINUE                                                          MASS  10
      DO 2 I1=1,MF                                                      MASS  11
      II=I1-1                                                           MASS  12
      AN(II+1,II+1)=1.0-X*Y-3*X*X-3*Y*Y+3*X*Y*Y+3*X*X*Y+2*X*X*X+2*Y*Y*Y-MASS  13
     12*X*X*X*Y-2*X*Y*Y*Y                                               MASS  14
      AN(II+1,II+4)=AX*(X-X*Y-2*X*X+2*X*X*Y+X*X*X-X*X*X*Y)              MASS  15
      AN(II+1,II+7)=AY*(Y-X*Y-2*Y*Y+2*X*Y*Y+Y*Y*Y-X*Y*Y*Y)             MASS  16
      AN(II+1,II+10)=X*Y+3*X*X-3*X*X*Y-3*X*Y*Y-2*X*X*X+2*X*X*X*Y+2*X*Y*YMASS  17
     1*Y                                                                MASS  18
      AN(II+1,II+13)=AX*(-X*X+X*X*Y+X*X*X-X*X*X*Y)                      MASS  19
      AN(II+1,II+16)=AY*(X*Y-2*X*Y*Y+X*Y*Y*Y)                           MASS  20
      AN(II+1,II+19)=-X*Y+3*X*X*Y+3*X*Y*Y-2*X*X*X*Y-2*X*Y*Y*Y           MASS  21
      AN(II+1,II+22)=AX*(-X*X*Y+X*X*X*Y)                                MASS  22
      AN(II+1,II+25)=AY*(-X*Y*Y+X*Y*Y*Y)                                MASS  23
      AN(II+1,II+28)=X*Y+3*Y*Y-3*X*X*Y-3*X*Y*Y-2*Y*Y*Y+2*X*X*X*Y+2*X*Y*YMASS  24
     1*Y                                                                MASS  25
      AN(II+1,II+31)=AX*(X*Y-2*X*X*Y+X*X*X*Y)                           MASS  26
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      AN(II+1,II+34)=AY*(-Y*Y+X*Y*Y+Y*Y*Y-X*Y*Y*Y)                    MASS  27
    2 CONTINUE                                                        MASS  28
      CALL GMTRA(AN,AS,MF,NF)                                         MASS  29
      CALL GMPRD(AS,AN,BF,NF,MF,NF)                                   MASS  30
      RETURN                                                          MASS  31
      END                                                             MASS  32
```



```
      DIMENSION AKK(36,36),AKL(36,36),AKM(36,36),AKN(36,36),A(78,78),RM(MAIN   1
     178,78),ARA(78,78),VU(78),AKF(108,108)                           MAIN   2
      INTEGER VC(30)                                                  MAIN   3
      EQUIVALENCE (AKF(1),ARA(1)),(RM(1),AKK(1))                      MAIN   4
C     ************************************************************MAIN   5
C     PROGRAM WRITTEN BY GEORGE GEORGOPOULOS                          MAIN   6
C     ************************************************************MAIN   7
C     AKK(NF,NF)                                                      MAIN   8
C     AKL(NF,NF)                                                      MAIN   9
C     AKM(NF,NF)                                                      MAIN  10
C     AKN(NF,NF)                                                      MAIN  11
C     A(N,N)                                                          MAIN  12
C     RM(N,N)                                                         MAIN  13
C     ARA(N,N)                                                        MAIN  14
C     VU(N)                                                           MAIN  15
C     AKF(NPK,NPK)                                                    MAIN  16
C     VC(NBC)                                                         MAIN  17
C     NF=DEGREES OF FREEDOM PER ELEMENT                               MAIN  18
C     NM=NUMBER OF ELEMENTS IN X-DIRECTION (WIDTH)                    MAIN  19
C     NELEM=NUMBER OF ELEMENTS                                        MAIN  20
```

```
C       NPOIN=NUMBER OF NODAL POINTS                                          MAIN  21
C       NFREE=DEGREES OF FREEDOM PER NODAL POINT                              MAIN  22
C       NPK=NPOIN*NFREE                                                       MAIN  23
C       NBC=NUMBER OF BOUNDARY CONTITIONS                                     MAIN  24
C       N=NPK-NBC                                                             MAIN  25
C       AX=LENGTH OF THE ELEMENT IN X-DIRECTION (IN)                          MAIN  26
C       AY=LENGTH OF THE ELEMENT IN Y-DIRECTION (IN)                          MAIN  27
C       E=MODULUS OF ELASTICITY (LBF/IN**2)                                   MAIN  28
C       RHO=DENSITY (LBM/IN**3)                                               MAIN  29
C       POISS=POISSON'S RATIO                                                 MAIN  30
C       THICK=THICKNESS (IN)                                                  MAIN  31
C       AL=SHELL'S OPENNING LENGTH (IN)                                       MAIN  32
        AX=0.75                                                               MAIN  33
        AY=1.542                                                              MAIN  34
        E=10300000                                                            MAIN  35
        RHO=0.098                                                             MAIN  36
        RHO=RHO/386.4                                                         MAIN  37
        POISS=0.334                                                           MAIN  38
        THICK=0.4                                                             MAIN  39
        AL=6.60                                                               MAIN  40
C       OMEGA=NATURAL FREQUENCY (RAD/SEC)                                     MAIN  41
C       FREQU=NATURAL FREQUENCY (CYCLES/SEC)                                  MAIN  42
C       COEF1=OMEGA*D1,FREQUENCY COEFFICIENT                                  MAIN  43
C       IF COORDINATE TRANSFORMATION IS DESIRED THEN AKL(I,J) IS THE          MAIN  44
C       DIRECTIONAL COSINE TRANSFORMATION MATRIX,THAT IS K'=(AKL)T*K*(AKL)     MAIN  45
C       D=E/(RHO*2*(1.0-POISS*POISS))                                         MAIN  46
C       OMEGA=SQRT(D/VU(I))                                                   MAIN  47
C       COEF1=OMEGA*D1                                                        MAIN  48
        NELEM=6                                                               MAIN  49
        NPOIN=12                                                              MAIN  50
        NBC=30                                                                MAIN  51
        NM=2                                                                  MAIN  52
        NM1=1                                                                 MAIN  53
        NM2=NM                                                                MAIN  54
        NFREE=9                                                               MAIN  55
        NPK=NFREE*NPOIN                                                       MAIN  56
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```
      NF=4*NFREE                                                        MAIN   57
      N=NPK-NBC                                                         MAIN   58
      NN=N                                                              MAIN   59
      D=E/(RHO*2*(1.0-POISS*POISS))                                     MAIN   60
      D1=AL*AL/THICK*SQRT((RHO*12)/E)                                   MAIN   61
      DO 1 I1=1,NF                                                      MAIN   62
      DO 1 J1=1,NF                                                      MAIN   63
      AKL(I1,J1)=0                                                      MAIN   64
    1 CONTINUE                                                          MAIN   65
      NAK=0                                                             MAIN   66
    2 NAK=NAK+1                                                         MAIN   67
      DO 3 I1=1,NPK                                                     MAIN   68
      DO 3 J1=1,NPK                                                     MAIN   69
      AKF(I1,J1)=0                                                      MAIN   70
    3 CONTINUE                                                          MAIN   71
      NJ=NELEM/NM                                                       MAIN   72
      DO 43 I=1,NJ                                                      MAIN   73
      IF (NAK.EQ.1) GO TO 5                                             MAIN   74
C     INSERT MASS 'M'                                                   MAIN   75
      READ (8,4) (((AKK(I1,J1),J1=1,NF),I1=1,NF))                       MAIN   76
    4 FORMAT (/,4E15.7)                                                 MAIN   77
    5 IF (NAK.GT.1) GO TO 6                                             MAIN   78
C     INSERT 'STIFFNESS' MATRIX                                         MAIN   79
      READ (8,4) (((AKK(I1,J1),J1=1,NF),I1=1,NF))                       MAIN   80
    6 DO 43 IR=1,NM1                                                    MAIN   81
      IC=0                                                              MAIN   82
    7 DO 9 JD=1,2                                                       MAIN   83
      DO 8 IJK=1,NFREE                                                  MAIN   84
      AKL(IJK+IC,IJK+IC)=1                                              MAIN   85
    8 CONTINUE                                                          MAIN   86
      IC=IC+NFREE                                                       MAIN   87
    9 CONTINUE                                                          MAIN   88
      IF (IC.EQ.(2*NFREE)) GO TO 7                                      MAIN   89
      CALL GTPRD(AKL,AKK,AKN,NF,NF,NF)                                  MAIN   90
      CALL GMPRD(AKM,AKL,AKN,NF,NF,NF)                                  MAIN   91
      DO 43 IP=1,NM2                                                    MAIN   92
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```
      READ (1,10) L,J,K,M                                              MAIN   93
   10 FORMAT (4I5)                                                     MAIN   94
      LL=L*NFREE                                                       MAIN   95
      JJ=J*NFREE                                                       MAIN   96
      KK=K*NFREE                                                       MAIN   97
      MM=M*NFREE                                                       MAIN   98
      LLL=LL-NFREE+1                                                   MAIN   99
      JJJ=JJ-NFREE+1                                                   MAIN  100
      KKK=KK-NFREE+1                                                   MAIN  101
      MMM=MM-NFREE+1                                                   MAIN  102
      L1=1                                                             MAIN  103
      DO 12 I1=LLL,LL                                                  MAIN  104
      M1=1                                                             MAIN  105
      DO 11 J1=LLL,LL                                                  MAIN  106
      AKF(I1,J1)=AKF(I1,J1)+AKN(L1,M1)                                 MAIN  107
      M1=M1+1                                                          MAIN  108
   11 CONTINUE                                                         MAIN  109
      L1=L1+1                                                          MAIN  110
   12 CONTINUE                                                         MAIN  111
      L1=1                                                             MAIN  112
      DO 14 I1=LLL,LL                                                  MAIN  113
      M1=NFREE+1                                                       MAIN  114
      DO 13 J1=JJJ,JJ                                                  MAIN  115
      AKF(I1,J1)=AKF(I1,J1)+AKN(L1,M1)                                 MAIN  116
      M1=M1+1                                                          MAIN  117
   13 CONTINUE                                                         MAIN  118
      L1=L1+1                                                          MAIN  119
   14 CONTINUE                                                         MAIN  120
      L1=1                                                             MAIN  121
      DO 16 I1=LLL,LL                                                  MAIN  122
      M1=2*NFREE+1                                                     MAIN  123
      DO 15 J1=KKK,KK                                                  MAIN  124
      AKF(I1,J1)=AKF(I1,J1)+AKN(L1,M1)                                 MAIN  125
      M1=M1+1                                                          MAIN  126
   15 CONTINUE                                                         MAIN  127
      L1=L1+1                                                          MAIN  128
```

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16 CONTINUE                                                    MAIN 129
   L1=1                                                        MAIN 130
   DO 18 I1=LLL,LL                                             MAIN 131
   M1=3*NFREE+1                                                MAIN 132
   DO 17 J1=MMM,MM                                             MAIN 133
   AKF(I1,J1)=AKF(I1,J1)+AKN(L1,M1)                            MAIN 134
   M1=M1+1                                                     MAIN 135
17 CONTINUE                                                    MAIN 136
   L1=L1+1                                                     MAIN 137
18 CONTINUE                                                    MAIN 138
   L1=NFREE+1                                                  MAIN 139
   DO 20 I1=JJJ,JJ                                             MAIN 140
   M1=1                                                        MAIN 141
   DO 19 J1=LLL,LL                                             MAIN 142
   AKF(I1,J1)=AKF(I1,J1)+AKN(L1,M1)                            MAIN 143
   M1=M1+1                                                     MAIN 144
19 CONTINUE                                                    MAIN 145
   L1=L1+1                                                     MAIN 146
20 CONTINUE                                                    MAIN 147
   L1=NFREE+1                                                  MAIN 148
   DO 22 I1=JJJ,JJ                                             MAIN 149
   M1=NFREE+1                                                  MAIN 150
   DO 21 J1=JJJ,JJ                                             MAIN 151
   AKF(I1,J1)=AKF(I1,J1)+AKN(L1,M1)                            MAIN 152
   M1=M1+1                                                     MAIN 153
21 CONTINUE                                                    MAIN 154
   L1=L1+1                                                     MAIN 155
22 CONTINUE                                                    MAIN 156
   L1=NFREE+1                                                  MAIN 157
   DO 24 I1=JJJ,JJ                                             MAIN 158
   M1=2*NFREE+1                                                MAIN 159
   DO 23 J1=KKK,KK                                             MAIN 160
   AKF(I1,J1)=AKF(I1,J1)+AKN(L1,M1)                            MAIN 161
   M1=M1+1                                                     MAIN 162
23 CONTINUE                                                    MAIN 163
   L1=L1+1                                                     MAIN 164
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24 CONTINUE                                                          MAIN 165
   L1=NFREE+1                                                        MAIN 166
   DO 26 I1=JJJ,JJ                                                   MAIN 167
   M1=3*NFREE+1                                                      MAIN 168
   DO 25 J1=MMM,MM                                                   MAIN 169
   AKF(I1,J1)=AKF(I1,J1)+AKN(L1,M1)                                  MAIN 170
   M1=M1+1                                                           MAIN 171
25 CONTINUE                                                          MAIN 172
   L1=L1+1                                                           MAIN 173
26 CONTINUE                                                          MAIN 174
   L1=2*NFREE+1                                                      MAIN 175
   DO 28 I1=KKK,KK                                                   MAIN 176
   M1=1                                                              MAIN 177
   DO 27 J1=LLL,LL                                                   MAIN 178
   AKF(I1,J1)=AKF(I1,J1)+AKN(L1,M1)                                  MAIN 179
   M1=M1+1                                                           MAIN 180
27 CONTINUE                                                          MAIN 181
   L1=L1+1                                                           MAIN 182
28 CONTINUE                                                          MAIN 183
   L1=2*NFREE+1                                                      MAIN 184
   DO 30 I1=KKK,KK                                                   MAIN 185
   M1=NFREE+1                                                        MAIN 186
   DO 29 J1=JJJ,JJ                                                   MAIN 187
   AKF(I1,J1)=AKF(I1,J1)+AKN(L1,M1)                                  MAIN 188
   M1=M1+1                                                           MAIN 189
29 CONTINUE                                                          MAIN 190
   L1=L1+1                                                           MAIN 191
30 CONTINUE                                                          MAIN 192
   L1=2*NFREE+1                                                      MAIN 193
   DO 32 I1=KKK,KK                                                   MAIN 194
   M1=2*NFREE+1                                                      MAIN 195
   DO 31 J1=KKK,KK                                                   MAIN 196
   AKF(I1,J1)=AKF(I1,J1)+AKN(L1,M1)                                  MAIN 197
   M1=M1+1                                                           MAIN 198
31 CONTINUE                                                          MAIN 199
   L1=L1+1                                                           MAIN 200
```

```
32 CONTINUE                                                    MAIN 201
   L1=2*NFREE+1                                                MAIN 202
   DO 34 I1=KKK,KK                                             MAIN 203
   M1=3*NFREE+1                                                MAIN 204
   DO 33 J1=MMM,MM                                             MAIN 205
   AKF(I1,J1)=AKF(I1,J1)+AKN(L1,M1)                            MAIN 206
   M1=M1+1                                                     MAIN 207
33 CONTINUE                                                    MAIN 208
   L1=L1+1                                                     MAIN 209
34 CONTINUE                                                    MAIN 210
   L1=3*NFREE+1                                                MAIN 211
   DO 36 I1=MMM,MM                                             MAIN 212
   M1=1                                                        MAIN 213
   DO 35 J1=LLL,LL                                             MAIN 214
   AKF(I1,J1)=AKF(I1,J1)+AKN(L1,M1)                            MAIN 215
   M1=M1+1                                                     MAIN 216
35 CONTINUE                                                    MAIN 217
   L1=L1+1                                                     MAIN 218
36 CONTINUE                                                    MAIN 219
   L1=3*NFREE+1                                                MAIN 220
   DO 38 I1=MMM,MM                                             MAIN 221
   M1=NFREE+1                                                  MAIN 222
   DO 37 J1=JJJ,JJ                                             MAIN 223
   AKF(I1,J1)=AKF(I1,J1)+AKN(L1,M1)                            MAIN 224
   M1=M1+1                                                     MAIN 225
37 CONTINUE                                                    MAIN 226
   L1=L1+1                                                     MAIN 227
38 CONTINUE                                                    MAIN 228
   L1=3*NFREE+1                                                MAIN 229
   DO 40 I1=MMM,MM                                             MAIN 230
   M1=2*NFREE+1                                                MAIN 231
   DO 39 J1=KKK,KK                                             MAIN 232
   AKF(I1,J1)=AKF(I1,J1)+AKN(L1,M1)                            MAIN 233
   M1=M1+1                                                     MAIN 234
39 CONTINUE                                                    MAIN 235
   L1=L1+1                                                     MAIN 236
```

```
40 CONTINUE                                                    MAIN 237
   L1=3*NFREE+1                                                MAIN 238
   DO 42 I1=MMM,MM                                             MAIN 239
   M1=3*NFREE+1                                                MAIN 240
   DO 41 J1=MMM,MM                                             MAIN 241
   AKF(I1,J1)=AKF(I1,J1)+AKN(L1,M1)                            MAIN 242
   M1=M1+1                                                     MAIN 243
41 CONTINUE                                                    MAIN 244
   L1=L1+1                                                     MAIN 245
42 CONTINUE                                                    MAIN 246
43 CONTINUE                                                    MAIN 247
   IF (NAK.GT.1) GO TO 45                                      MAIN 248
   READ (1,44) (VC(I1),I1=1,NBC)                               MAIN 249
44 FORMAT (8I10)                                               MAIN 250
45 DO 46 I1=1,NBC                                              MAIN 251
   LA=VC(I1)                                                   MAIN 252
   IN=NPK-LA-I1+1                                              MAIN 253
   IF (IN.LE.0) GO TO 46                                       MAIN 254
   DO 46 J1=1,IN                                               MAIN 255
   LB=LA+1                                                     MAIN 256
   CALL RINT(AKF,NPK,NPK,LA,LB)                                MAIN 257
   CALL CINT(AKF,NPK,LA,LB)                                    MAIN 258
   LA=LB                                                       MAIN 259
46 CONTINUE                                                    MAIN 260
   IF (NAK.GT.1) GO TO 48                                      MAIN 261
   DO 47 I1=1,N                                                MAIN 262
   DO 47 J1=1,N                                                MAIN 263
   A(I1,J1)=AKF(I1,J1)                                         MAIN 264
47 CONTINUE                                                    MAIN 265
   NM=NELEM                                                    MAIN 266
   NM1=1                                                       MAIN 267
   NM2=NELEM                                                   MAIN 268
   GO TO 2                                                     MAIN 269
48 DO 49 I1=1,N                                                MAIN 270
   DO 49 J1=1,N                                                MAIN 271
   RM(I1,J1)=AKF(I1,J1)                                        MAIN 272
```

```
49 CONTINUE                                                              MAIN 273
   WRITE (3,55)                                                          MAIN 274
   CALL NROOT(N,RM,A,VU,ARA)                                             MAIN 275
   DO 50 I=1,NN                                                          MAIN 276
   OMEGA=SQRT(D/VU(I))                                                   MAIN 277
   FREQU=OMEGA/6.28                                                      MAIN 278
   COEF1=OMEGA*D1                                                        MAIN 279
   WRITE (3,53) I,VU(I),OMEGA,FREQU,COEF1                                MAIN 280
50 CONTINUE                                                              MAIN 281
   WRITE (3,55)                                                          MAIN 282
   READ (1,44) (VC(I1),I1=1,NBC)                                         MAIN 283
   A1=0                                                                  MAIN 284
   I1=1                                                                  MAIN 285
   I2=1                                                                  MAIN 286
   DO 52 I=1,NPK                                                         MAIN 287
   IF ((VC(I1)-I).EQ.0) GO TO 51                                         MAIN 288
   WRITE (3,54) I,(ARA(I2,J2),J2=1,6)                                    MAIN 289
   I2=I2+1                                                               MAIN 290
   GO TO 52                                                              MAIN 291
51 WRITE (3,54) I,A1,A1,A1,A1,A1,A1                                      MAIN 292
   I1=I1+1                                                               MAIN 293
52 CONTINUE                                                              MAIN 294
53 FORMAT (1X,'I=',I3,2X,'VU(I)=',E13.6,2X,'OMEGA=',E13.6,2X,'FREQU='    MAIN 295
  1,E13.6,2X,'COEF1=',E13.6,2X,/)                                        MAIN 296
54 FORMAT (1X,'I=',I3,6E20.6,/)                                          MAIN 297
55 FORMAT (1H1)                                                          MAIN 298
   CALL EXIT                                                             MAIN 299
   END                                                                   MAIN 300
```

```
= 1   VU(I)= 0.?6543?E 01   OMEGA= 0.6??4??E 04   FREQU= 0.129525E 04   COEF1= 0.152268E 02
= 2   VU(I)= 0.??????E 01   OMEGA= 0.47?3?0E 05   FREQU= 0.6??3?4E 04   COEF1= 0.7?6874E 02
= 3   VU(I)= 0.66????E 01   OMEGA= 0.6??44?E 05   FREQU= 0.??4?71E 04   COEF1= 0.111099E 03
= 4   VU(I)= 0.??6?9?E 01   OMEGA= 0.80?077E 05   FREQU= 0.122022E 05   COEF1= 0.150500E 03
= 5   VU(I)= 0.?45???E 01   OMEGA= 0.9657?2E 05   FREQU= 0.153789E 05   COEF1= 0.180791E 03
= 6   VU(I)= 0.??0596E 01   OMEGA= 0.995562E 05   FREQU= 0.158529E 05   COEF1= 0.196?64E 03
= 7   VU(I)= 0.1?60?7E 01   OMEGA= 0.115?71E 06   FREQU= 0.184668E 05   COEF1= 0.217092E 03
= 8   VU(I)= 0.1070??E 01   OMEGA= 0.146130E 06   FREQU= 0.232691E 05   COEF1= 0.273547E 03
= 9   VU(I)= 0.7582?4E 00   OMEGA= 0.173418E 06   FREQU= 0.276462E 05   COEF1= 0.325004E 03
= 10  VU(I)= 0.5???11E 00   OMEGA= 0.208013E 06   FREQU= 0.331231E 05   COEF1= 0.389390E 03
= 11  VU(I)= 0.520534E 00   OMEGA= 0.209541E 06   FREQU= 0.333664E 05   COEF1= 0.392250E 03
= 12  VU(I)= 0.490856E 00   OMEGA= 0.215783E 06   FREQU= 0.343603E 05   COEF1= 0.403934E 03
= 13  VU(I)= 0.451010E 00   OMEGA= 0.225113E 06   FREQU= 0.358460E 05   COEF1= 0.421400E 03
= 14  VU(I)= 0.327986E 00   OMEGA= 0.263977E 06   FREQU= 0.420346E 05   COEF1= 0.494152E 03
= 15  VU(I)= 0.?15?14E 00   OMEGA= 0.268974E 06   FREQU= 0.428302E 05   COEF1= 0.503505E 03
= 16  VU(I)= 0.282446E 00   OMEGA= 0.283961E 06   FREQU= 0.452167E 05   COEF1= 0.531560E 03
= 17  VU(I)= 0.2614?3E 00   OMEGA= 0.301509E 06   FREQU= 0.480109E 05   COEF1= 0.564409E 03
= 18  VU(I)= 0.260022E 00   OMEGA= 0.302947E 06   FREQU= 0.482400E 05   COEF1= 0.567102E 03
= 19  VU(I)= 0.232757E 00   OMEGA= 0.313352E 06   FREQU= 0.498969E 05   COEF1= 0.586580E 03
= 20  VU(I)= 0.197946E 00   OMEGA= 0.339799E 06   FREQU= 0.541079E 05   COEF1= 0.636083E 03
= 21  VU(I)= 0.187271E 00   OMEGA= 0.349348E 06   FREQU= 0.556287E 05   COEF1= 0.653962E 03
= 22  VU(I)= 0.160674E 00   OMEGA= 0.377156E 06   FREQU= 0.600569E 05   COEF1= 0.706017E 03
= 23  VU(I)= 0.142852E 00   OMEGA= 0.398597E 06   FREQU= 0.634709E 05   COEF1= 0.746153E 03
= 24  VU(I)= 0.133782E 00   OMEGA= 0.413328E 06   FREQU= 0.658166E 05   COEF1= 0.773729E 03
```

I= 1	-0.511354E-01	0.144298E-02	-0.461120E-01	0.304674E-02	0.134910
I= 2	0.0	0.0	0.0	0.0	0.0
I= 3	-0.244746E 00	0.260875E 00	0.269269E 00	-0.329109E 00	-0.167391
I= 4	0.701006E-03	0.624103E-03	-0.285640E-01	0.235794E-01	0.796182
I= 5	0.0	0.0	0.0	0.0	0.0
I= 6	-0.326231E-01	0.794584E-01	0.142047E-01	-0.510628E 00	-0.268741
I= 7	0.147719E-03	-0.984164E-03	-0.219148E-03	0.102991E-04	0.771893
I= 8	0.713418E-01	-0.607490E-01	0.274764E-01	0.698096E-03	0.122499
I= 9	0.0	0.0	0.0	0.0	0.0
I= 10	0.689734E-04	0.111425E-02	-0.442602E-01	0.243552E-02	0.515433
I= 11	0.101314E 00	-0.372633E-01	0.469012E-01	0.636452E-02	0.191855
I= 12	-0.207793E 00	-0.138350E 00	0.195061E 00	-0.250002E 00	0.119202
I= 13	0.461195E-03	0.974450E-03	-0.273910E-01	0.196465E-01	-0.254577
I= 14	0.144975E-03	-0.105195E-02	-0.255779E-02	0.351467E-02	0.117836
I= 15	-0.170369E-01	-0.334045E-01	-0.122244E-01	-0.296785E 00	0.230981
I= 16	0.309008E-03	-0.875294E-03	0.225516E-02	-0.217588E-02	-0.930223
I= 17	0.496620E-01	0.334711E-01	0.242667E-01	0.452345E-02	0.442079
I= 18	0.169253E 00	-0.309753E 00	-0.211268E-01	0.102541E 00	0.225245
I= 19	0.170692E-01	0.191035E-02	-0.392087E-01	-0.264074E-02	-0.427285
I= 20	0.121807E 00	0.536730E-01	0.786961E-01	0.851674E-02	0.145415
I= 21	0.971179E-01	-0.171568E 00	0.242556E 00	-0.941773E-01	0.264505
I= 22	-0.318789E-02	0.135632E-02	-0.244810E-01	0.999245E-02	-0.239466
I= 23	0.206004E-01	0.352534E-03	-0.264586E-02	0.436145E-02	0.277955
I= 24	0.137790E-01	-0.414250E-01	-0.721484E-02	-0.149507E 00	0.338730
I= 25	0.410401E-03	0.138251E-02	0.271372E-02	-0.335539E-02	-0.133.4.

I= 26	-0 269[illegible]E-01	0.438[illegible]E-01	-0.222350E-02	-0.212482E-02	-0.9[illegible]
I= 27	0 180980E 00	0 277873E 00	0.948083E-01	0.102330E 00	0.763863
I= 28	0.616206E-05	0.239645E-02	-0.358355E-01	-0.607156E-02	-0.121840
I= 29	0 0	0.0	0.0	0.0	0.0
I= 30	0.263605E 00	0.220764E 00	0.340115E 00	-0.1.0071E-01	0.262359
I= 31	-0.946964E-01	0.123943E-02	-0.226793E-01	0.489387E-02	-0.283338
I= 32	0 0	0.0	0.0	0.0	0.0
I= 33	0.275546E-01	0.652267E-01	0.571091E-02	-0.252392E-01	0.411635
I= 34	0.197993E-03	0.140997E-02	-0.229975E-03	-0.963913E-04	-0.334077
I= 35	-0.126700E 00	-0.127073E 00	-0.110604E 00	-0.758896E-02	-0.242965
I= 36	0.0	0.0	0.0	0.0	0.0
I= 37	-0 157478E-03	0.547172E-03	-0.237328E-01	-0.507328E-02	0.812023
I= 38	0 0	0.0	0.0	0.0	0.0
I= 39	-0 328822E 00	0.231437E 00	0.263573E 00	0.325859E-01	0.259532
I= 40	-0 154778E-03	0.108153E-02	-0.310097E-01	-0.251949E-03	0.97176[illegible]
I= 41	0.0	0.0	0.0	0.0	0.0
I= 42	-0.100789E-01	0.151346E-01	0.192275E-02	-0.413999E 00	-0.22343[illegible]
I= 43	-0 415609E-04	0.773829E-04	0.145939E-04	0.255080E-03	-0.89694[illegible]
I= 44	0 709324E-01	-0.578107E-01	0.231067E-01	-0.189896E-02	-0.9990[illegible]
I= 45	0 0	0.0	0.0	0.0	0.0
I= 46	-0 284919E-01	0.574603E-03	-0.226980E-01	-0.435286E-02	0.52003
I= 47	0 100767E 00	-0.358814E-01	0.425558E-01	-0.290310E-0[illegible]	-0.72031
I= 48	-0 192703E 00	-0.127046E 00	0.109738E 00	0.244043E-01	-0.96514
I= 49	-0 252346E-01	0.664313E-02	-0 257439E-01	-0.223622E-02	0.35977
I= 50	[illegible]	-0 109222E-02	[illegible]	0.[illegible]E-0[illegible]	0.439[illegible]

32

I= 52	0.120516E-03	-0.202490E-03	0.126328E-02	-0.387125E-03	-0.40417
I= 53	0.492510E-01	0.320986E-01	0.345912E-01	-0.157288E-02	0.74848
I= 54	0.163376E 00	-0.292280E 00	-0.272545E-01	-0.964546E-02	-0.19216
I= 55	0.165201E-03	0.971168E-03	-0.200715E-01	-0.524774E-02	0.43189
I= 56	0.121068E 00	0.528291E-01	0.803750E-01	-0.313767E-02	-0.54992
I= 57	0.914364E-01	-0.155985E 00	0.266885E 00	0.918327E-02	-0.16996
I= 58	0.202765E-03	0.122618E-02	-0.263489E-01	-0.293461E-02	-0.52020
I= 59	0.109782E-02	0.126573E-02	-0.131743E-02	0.192245E-01	0.35582
I= 60	0.329144E-02	-0.832931E-02	-0.426090E-02	-0.116916E 00	0.32707
I= 61	0.167231E-03	0.696171E-03	0.192123E-02	-0.125956E-02	0.18265
I= 62	-0.269143E-01	0.464457E-01	-0.279500E-02	0.120569E-02	0.11390
I= 63	0.175424E 00	0.265502E 00	0.831211E-01	-0.804446E-02	-0.34914
I= 64	0.107482E-02	0.866874E-02	-0.183520E-01	-0.543865E-02	0.23132
I= 65	0.0	0.0	0.0	0.0	0.0
I= 66	0.252026E 00	0.199793E 00	0.339934E 00	0.152933E-02	-0.29057
I= 67	0.173189E-02	0.165521E-02	-0.240014E-01	-0.474317E-02	-0.47677
I= 68	0.0	0.0	0.0	0.0	0.0
I= 69	0.650660E-02	0.717448E-02	-0.252499E-02	-0.149776E-01	0.32514
I= 70	-0.103572E-03	-0.276675E-03	-0.707407E-04	-0.252719E-02	-0.11291
I= 71	-0.134900E 00	-0.120816E 00	-0.111674E 00	0.315189E-02	0.10519
I= 72	0.0	0.0	0.0	0.0	0.0
I= 73	0.0	0.0	0.0	0.0	0.0
I= 74	0.0	0.0	0.0	0.0	0.0
I= 75	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

I= 77	0 0	0.0	0.0	0.0	0.0
I= 78	0 0	0.0	0.0	0.0	0.0
I= 79	0.0	0.0	0.0	0.0	0.0
I= 80	0.7071975-01	-0.565680E-01	0.207340E-01	-0.430358E-02	-0.22495
I= 81	0 0	0.0	0.0	0.0	0.0
I= 82	0.0	0.0	0.0	0.0	0.0
I= 83	0 100407E 00	-0.751678E-01	0.491713E-01	-0.967730E-02	-0.26257
I= 84	-0.191724E 00	-0.125808E 00	0.200774E 00	0.152604E 00	-0.19616
I= 85	-0 619427E-04	0.846033E-03	-0.205000E-01	-0.879939E-02	0.82407
I= 86	-0.739467E-05	0.111222E-02	0.164186E-03	-0.562933E-03	-0.19300
I= 87	0 0	0.0	0.0	0.0	0.0
I= 88	0.0	0.0	0.0	0.0	0.0
I= 89	0 490570E-01	0.317931E-01	0.247159E-01	-0.553733E-02	-0.42569
I= 90	0 160474E 00	-0.286519E 00	-0 256677E-01	-0.724159E-01	-0.17077
I= 91	0.0	0 0	0.0	0.0	0.0
I= 92	0.120628E 00	0.523582E-01	0.308527E-01	-0.113754E-01	-0.20276
I= 93	0 904430E-01	-0.153317E 00	0.268577E 00	0.565876E-01	-0.15300
I= 94	0.240195E-01	0.146720E-02	-0.269670E-01	-0.938604E-02	0.88437
I= 95	-0 273229E-01	-0.112404E-02	0.472213E-04	-0.153076E-02	-0.61805
I= 96	0.0	0.0	0.0	0.0	0.0
I= 97	0 0	0.0	0.0	0.0	0.0
I= 98	-0 268564E-01	0.456509E-01	-0.303173E-02	0.349947E-02	0.49849
I= 99	0 174229E 00	0.261630E 00	0.827827E-01	-0.695375E-01	-0.35691
I=100	0 0	0.0	0.0	0.0	0.0

I=101	0.0	0.0	0.0	0.0	0.0
I=102	0.2500475 00	0.1980695 00	0.3412015 00	0.652000E-02	-0.162616
I=103	0.1111425-02	0.8850355-03	-0.2472055-01	-0.104523E-01	0.705863
I=104	0.0	0.0	0.0	0.0	0.0
I=105	0.0	0.0	0.0	0.0	0.0
I=106	0.0	0.0	0.0	0.0	0.0
I=107	-0.1342245 00	-0.1190465 00	-0.1120715 00	0.1030875-01	0.32270
I=108	0.0	0.0	0.0	0.0	0.0